Mansoura University	Facultyof Engineering	Electronics & Comm. Dept.	
Mid-Term Exam–4th-Year		Total Marks [30]	
28/11/2015	Optical Communication Systems	4 pages	Time: 1.5 Hour
Name		Sec	

Q_1. Estimate one of the following relations **[5 marks]**

- sec(2) Energy values for Hydrogen atom by solving Schrödinger equation
- sec(4) The density of states $\rho_{c,v}(E)$ for bulk semiconductor materials

Solution of Q_1

$$E = E_K + E_P$$

$$= \frac{1}{2}mv^2 + \left(\frac{-e^2}{4\pi\varepsilon_0 r}\right)$$

Solution of schrodinger:

$$\psi(x) = e^{jkx}$$

↳ The Function is periodic every (L)

$$\therefore \psi(x) = \psi(x+L)$$

$$e^{jkx} = e^{jkx} \cdot e^{jkL}$$

$$\therefore kL = n(2\pi) \qquad n = 1, 2, 3, \ldots$$

$$\boxed{k = \frac{2\pi n}{L}}$$

$$\because L = 2\pi r \rightarrow \boxed{K = \frac{n}{r}} \rightarrow (1)$$

↳ De-Broglie Relation:

$$P = \hbar K = \frac{\hbar n}{r}$$

$$pr = \hbar n$$

$$mv \cdot r = \hbar n$$

$$v = \frac{\hbar n}{mr} \longrightarrow (2)$$

↳ Coulmb's Force:

$$F = \frac{e^2}{4\pi\varepsilon r^2} \rightarrow (3)$$

$$F = m \cdot a = m\frac{v^2}{r}$$

$$V^2 = \frac{e^2}{4\pi\varepsilon_0 r m} \rightarrow (5)$$

Eqn (2) power 2: $V^2 = \frac{\hbar^2 n^2}{m^2 r^2}$

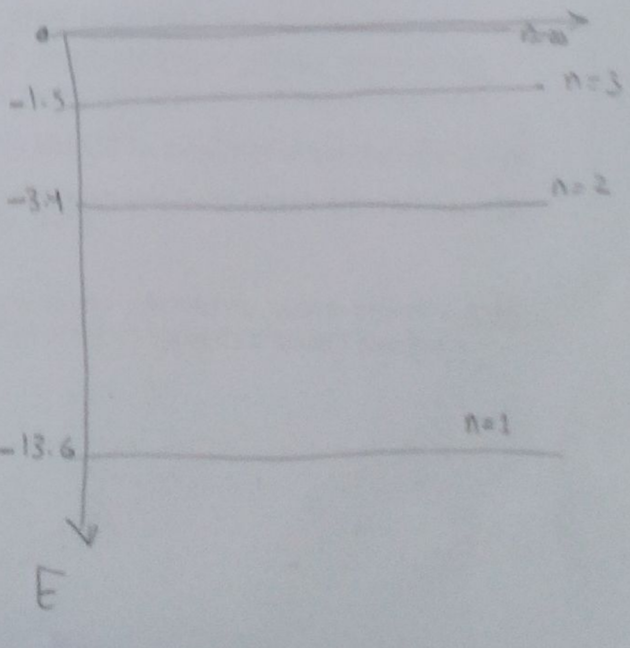
$$\therefore r = \frac{\hbar^2 n^2 \cdot 4\pi\varepsilon_0}{me^2} = \frac{h^2 n^2 \varepsilon_0}{\pi m e^2} \rightarrow (6)$$

$$\therefore \boxed{V = \frac{e^2}{2hn\varepsilon_0}}$$

$$E_K = \frac{1}{2}mv^2 = \frac{e^4 m}{8h^2n^2\varepsilon_0^2} \rightarrow \text{I}$$

$$E_p = \frac{-e^2}{4\varepsilon_0^2 h^2 n^2}$$

$$E = \frac{-e^4 m}{8h^2n^2\varepsilon_0^2} = \frac{-13.6}{n^2}\ \text{eV}$$



[8 marks]

Q2. Compare between

- Materials: $Al_{0.1}Ga_{0.9}As$ and $Al_{0.7}Ga_{0.3}As$ (direct energy bandgap E_g- type- application)
- Structures: N-GaAs/ N-$Al_{0.2}Ga_{0.8}As$, N-Si/ P-Si (Type of junction)

Solution of Q_2

Materials	Direct Energy Bandgap E_g	Type of Material Bandgap	Application
$Al_{0.1}Ga_{0.9}As$	$E_g = 1.424 + 1.247X$ 1.55	Direct	optical Sensor
$Al_{0.7}Ga_{0.3}As$	$E_g = 1.424 + 1.247X + 1.147(X-0.45)^2$ $= 2.37$	Indirect	photo detector

Structures	Junction
N-GaAs/ N-$Al_{0.2}Ga_{0.8}As$	~~Isotype~~ NN-HetroJunction
N-Si/ P-Si	Homo Junction

Q3. Cubic structure with 1mm side length and is made from intrinsic material with characteristics defined at room temperature 300 K as shown below. [17 marks]

- E_2 (energy value in the conduction band) $=E_c+\frac{\hbar^2(K-6)^2}{2m_c}$, E_c=-48.9 eV, m_c=0.98m_0 → conductance ↓
- E_1 (energy value in the valence band) $=E_v-\frac{\hbar^2 K^2}{2m_v}$, E_v=-50 eV, m_v=0.49m_0

a) Draw qualitatively the E-K diagram and determine the energy bandgap Eg and cutoff wavelength, comment on the mobility of carriers (μ_e and μ_h), determine whether the material has direct or indirect bandgap.

b) Draw qualitatively the density of state curves for conduction and valence bands and calculate the intrinsic carrier concentrations. Redraw qualitatively the density of state curves when the dimensions of the structure have been changed to 1mm×1mm×10A.

c) Calculate the probabilities of finding electrons at E=-48 eV and holes at E=-51 eV.

d) Calculate the Fermi level energy E_F. How to modify the Fermi level energy?

Hint: Electron charge, q=1.6×10^{-19} coulombs, m_0 = 9.1×10^{-31} kg, Boltzmann's constant K= 1.38×10^{-23} JK^{-1}, reduced Planck's constant $\hbar$=1.05×10^{-34} Js.

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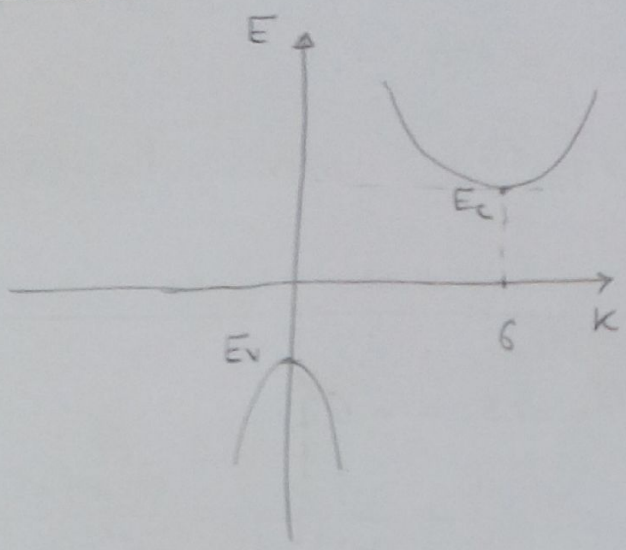
Solution of Q_3

E-K Diagram



Energy band-gap = $E_c - E_v = -48.9 - (-50)$

$= 1.1\ eV$

Cutoff wavelength = $\frac{1.24}{E_g} = \frac{1.24}{1.1} = 1.127\ \mu m$

Comment on carriers mobility $\mu \propto \frac{1}{m^*}$

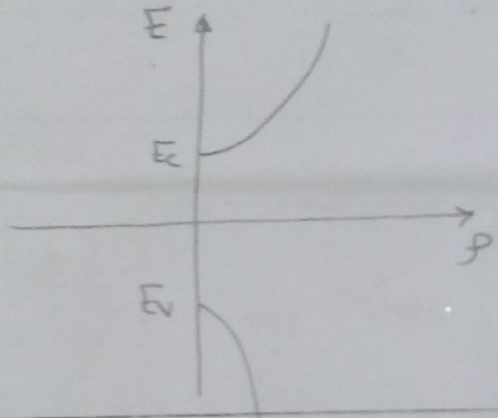
$m_c > m_v$

$\therefore$ Mobility of electrons in C.B < smaller than holes in V.B

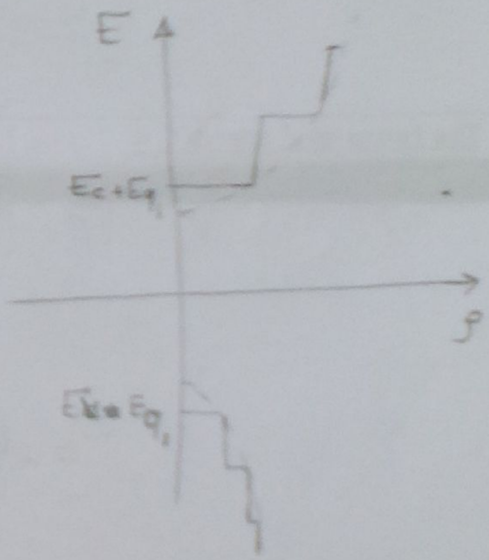
Determine whether the material has direct or indirect bandgap

Indirect BandGap

Density of states for 1mm×1mm×1mm structure



Density of states for 1mm×1mm×10A structure.



Intrinsic carrier concentration =

$$n_i = (N_c \cdot N_v)^{\frac{1}{2}}\ e^{-E_g/2KT}$$

$$N_c = 2\left(\frac{0.98*9.1*10^{-31}*1.38*10^{-23}*300}{2\pi\,(1.05*10^{-34})^2}\right)^{3/2}$$

$$= 2.4*10^{25}\ m^{-3}$$

$$N_v = 2\left(\frac{0.49*9.1*10^{-31}*1.38*10^{-23}*300}{2\pi\,(1.05*10^{-34})^2}\right)^{3/2} = 8.7*10^{24}\ m^{-3}$$

$$n_i = \sqrt{(8.7*10^{24})*2.4*10^{25}}\ .\ e^{-(1.1/0.025)}$$

$$= 4*10^{15}\ m^{-3}$$

$$= 4*10^{9}\ cm^{-3}$$

The probability of finding electrons at E=-48 eV =

$$E_f = \frac{E_c + E_v}{2} + \frac{3}{4} KT \ln\left(\frac{m_v}{m_c}\right)$$

$$(E - E_f)/KT = (-48 + 49.45) / 0.025 = 58$$

$$f(E) = \frac{1}{e^{(E-E_f)/KT} + 1} = e^{-58} = 6 * 10^{-26} \leftarrow \text{approximatly zero}$$

The probability of finding holes at E=-51 eV =

$$(E - E_f)/KT = (-51 + 49.45)/0.025 = -62$$

$$1 - f(E) = 1 - \frac{1}{e^{(E-E_f)/KT} + 1} = 1 - \frac{1}{e^{-62} + 1} = \frac{e^{-62} + 1 - 1}{e^{-62} + 1}$$

$$= \frac{e^{-62}}{e^{-62} + 1} \leftarrow \text{Approximatly zero}$$

The Fermi level energy E_f =

$$E_f = \frac{\overset{-48.9}{E_c} + \overset{-50}{E_v}}{2} + \frac{3}{4} \underset{1.38*10^{-23}}{K} \underset{300}{T} \ln\left(\frac{m_v}{m_c}\right) \quad 0.45,\ 0.98$$

$$= -49.45 - (2.15 * 10^{-21}) = -49.45$$

The Fremi level energy can be modified by .... Doping

Good Luck!

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